Jinglai Shen
N. Harris McClamroch
Department of Aerospace Engineering
The University of Michigan
Ann Arbor, MI 48109-2140

Translational and Rotational Maneuvers of an Underactuated Space Robot using Prismatic Actuators

Abstract

We study the simultaneous control of three dimensional translation and rotation of an underactuated multibody space robot using sliding masses that are configured as ideal prismatic actuators. A crucial assumption is that the total linear and angular momenta of the space robot are zero. The prismatic actuators may be intentional actuation devices or they may be dual-use devices such as retractable booms, tethers, or antennas that can also serve as space robot actuation devices. The paper focuses on the underactuation case, i.e., the space robot has three independent prismatic actuators, which are used to control the six base body degrees of freedom. Controllability results are developed, revealing controllability properties for the base body translation, base body attitude, and actuator displacement. Based on the controllability results, an algorithm for rest-to-rest base body maneuvers is constructed using a Lie bracket expansion. An example of a three dimensional space robot maneuver is presented. The results in the paper demonstrate the importance of "nonholonomy" and related nonlinear control approaches for space robots that satisfy the prismatic actuation assumptions.

KEY WORDS—space robots, translational and rotational maneuvers, nonlinear control, underactuated systems, prismatic actuators, nonholonomy

1. Introduction

Dynamics and control of complex multibody space robots for future space missions have attracted considerable attention. In space missions, the relative position and attitude of space robots often need to be accurately controlled over long time periods. This imposes severe requirements on control and system design to achieve high precision positioning and pointing while maintaining long duration flight and high system reliability. If the space robot has retractable booms, telescopes, or antennas, it may be possible to use such retractable prismatic devices as sole or auxiliary actuators to achieve high precision base body positioning and pointing. This paper demonstrates the feasibility of such an approach.

There is no prior published literature on the space robot positioning and pointing problem considered in this paper, but there is an extensive literature on related robotics problems. Complex space robots have been viewed as multibody mechanical systems consisting of a rigid base body and a number of movable components. The relative motions of these movable components with respect to the base body define the shape change of the robot. Traditional space robot control approaches often ignore coupling between the space robot translational dynamics, attitude dynamics, and shape dynamics, but recent progress in the dynamics and control of multibody space systems suggests that shape change mechanisms can be actively utilized for control purposes. See Senda et al. (1995), Spofford and Akin (1990), Ueba and Yasaka (1994), and Yoshida and Umetani (1990), where attitude control of free-flying space robots using shape change mechanisms is investigated. In Carignan and Akin (2000), the authors study the coupling between the base body dynamics and robot manipulator dynamics; they propose a scheme to reduce undesired base body motions when a manipulator performs a maneuver. This scheme is tested on an underwater robot. Other experimental studies include Koningstein and Cannon (1995) for a free-flying robot with two arms and Walsh and Sastry (1995) for a planar multi-link space robot. Attitude and shape control for fully actuated multibody space systems are treated in Umetani and Yoshida (1989) using inverse dynamics. In Matsumoto, Wakabayashi, and Watanbe (2000), internal shape change and gravity gradient effects are combined to control base body attitude.

A common assumption in the above studies, valid for many space robot systems, is that the total external force and moment on the entire space robot can be neglected so that the

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total spatial linear and angular momenta are conserved. In particular, the conservation of angular momentum provides a non-integrable constraint, i.e., a nonholonomic constraint in the sense of classical mechanics. Hence, the space robot can be viewed as a nonholonomic system. Noholonomic control systems have been widely studied in the robotics and nonlinear control communities in the last few years (Kolmanovsky and McClamroch 1995). One of the most interesting problems is the control of underactuated systems where the number of control inputs is less than the number of degrees of freedom to be controlled and the system is controllable in a nonlinear sense, but not in a linear sense. We only mention a few results here. In Kelly and Murray (1995), controllability results are presented for a class of robotic systems based on use of the Lie group structure. Motion planning algorithms are constructed for purely kinematic systems in Li and Cannay (1990), Leonard and Krishnaprasad (1995), Murray and Sastry (1993), and Murray, Li, and Sastry (1994). In Arai, Tanie, and Shiroma (1998) and Arai, Tanie, and Tachi (1993), controllability and feedback control problems are studied for planar underactuated manipulators. See Kolmanovsky and McClamroch (1995) and the references therein for a comprehensive survey of developments in this area.

Applications of nonlinear control theory to momentum-conserved underactuated multibody space robots have also been studied by many researchers. Krishnaprasad (1990) and Sreenath (1992) developed results on the attitude control of underactuated multibody space systems. Planar space robot reconfiguration has been investigated in Reyhanoglu and McClamroch (1992) and Walsh and Sastry (1995). A hybrid control scheme consisting of a continuous inner loop and a discrete outer loop is proposed in Giamberardino, Monaco, and Normand-Cyrot (2000) for planar maneuvers. Three dimensional attitude and shape maneuvers are even more challenging. There are substantial modeling and computational difficulties that are due to the complex dynamics of three dimensional rotations. This problem is treated in a unified formulation in Rui, Kolmanovsky, and McClamroch (2000), where controllability analysis and constructive motion planning are carried out and applied to several interesting shape change mechanisms.

In this paper, we address controllability and motion planning problems for three dimensional translational and rotational maneuvers of an underactuated space robot. Prismatic actuators, as described previously, are used to control the robot shape. As we shall show, these actuators are sufficient to control the position and attitude of the base body of the space robot. A planar version of this problem has been addressed in Shen and McClamroch (2001). Although prismatic actuators may only have limited translation control authority due to stroke limits, they can be used in place of thrusters for high precision local positioning and global attitude control. This leads to reduced fuel consumption and improves reliability in case of thruster failure.

The paper is organized as follows. In Section 2, we describe the configuration of a space robot with several prismatic actuators. The notation and key assumptions are introduced. In Section 3, equations of motion are derived based on the assumptions. Controllability properties are studied in Section 4. A necessary controllability condition for the minimum number and placement geometry of prismatic actuators is given. The remainder of Section 4 concentrates on nonlinear controllability properties of a space robot with three independent prismatic actuators. Controllability equivalence between base body translation, base body rotation and shape change is demonstrated. Local controllability is shown to imply "global" controllability, and its relation to the Lie algebraic rank condition is identified. Since the equations of motion are globally real analytic, it is further shown that if the Lie algebraic rank condition is satisfied at one shape, then there is a simple algorithm for solving rest-to-rest motion planning problems at different shapes. In Section 5, a design procedure for rest-to-rest base body maneuvers using three independent prismatic actuators is described. A three dimensional base body translational and rotational maneuver illustrates the controllability results and the maneuver design procedure.

2. Space Robot Description

We study an idealized space robot consisting of a rigid base body and several prismatic actuators (see Figure 1). The base body can translate and rotate in three dimensional space. The actuator mass elements can slide along straight slots that are fixed in the base body; the slot axes need not pass through the center of mass of the base body. These actuators may be robot booms or other space robot components that undergo constrained prismatic motions relative to the base body. For simplicity, the actuators are modeled as ideal mass particles controlled by electrical-mechanical linear motors, and they are referred to as prismatic actuators. We assume that in an appropriate inertial coordinate frame, the total external forces and moments on the system are zero. The total spatial linear and angular momenta of the space robot are further assumed to be zero.

The notation is:

m_b = the mass of the base body;

I_b = the moment of inertia of the base body with respect to the center of mass of the base body;

r = the inertial position vector of the center of mass of the base body;

R = the rotation matrix from the base body coordinates to the inertial frame;

m_i = the mass of the ith prismatic actuator;

z_i = the relative displacement of the ith actuator mass with respect to the base body;

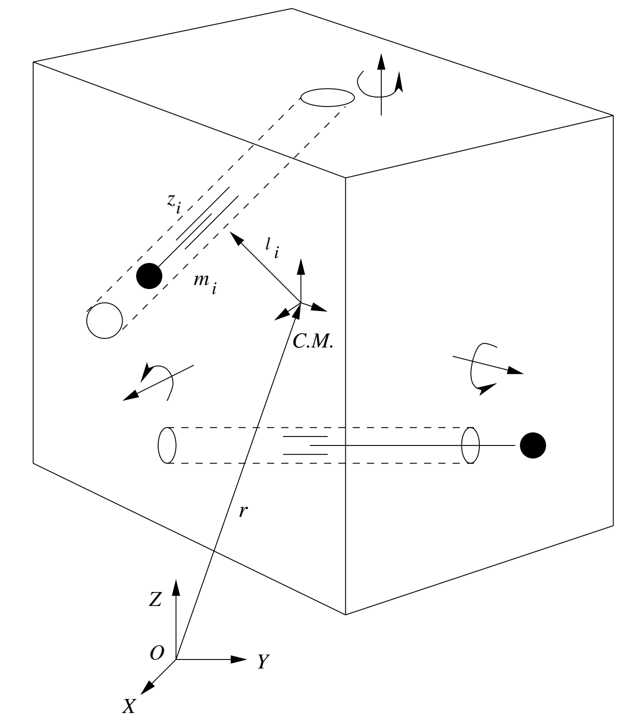

Fig. 1. Schematic configuration of a space robot with prismatic actuators.

l_i = the minimum distance from the center of mass of the base body to the ith slot;

R_i = the constant rotation matrix defining the orientation of the ith slot with respect to the base body frame.

Here $r \in \mathbb{R}^3$, $R \in \mathrm{SO}(3)$, $R_i \in \mathrm{SO}(3)$, $z_i \in \mathbb{R}$, and $z_i = 0$ corresponds to a position of the ith actuator mass such that its distance to the center of mass of the base body is minimum. Indeed this minimum distance is l_i.

The constant matrix R_i defines a transformation from the base body frame to the ith prismatic actuator frame defined as follows: its origin is located at the point in the slot whose distance to the center of mass of the base body is minimum, its X axis is aligned with the slot axis, its Y axis is aligned in the direction of the minimum distance to the center of mass of the base body, and its Z axis completes an orthogonal coordinate frame, following the right hand rotation rule. Therefore, $R_i(1)$, the first column of R_i, denotes the direction of the ith slot axis in the base body frame. Prismatic actuators are called independent if their slot axis directions are linearly independent.

3. Equations of Motion

We obtain the equations of motion for a space robot with n prismatic actuators in this section. For convenience, the origin of the inertial frame is chosen at the fixed center of mass of the space robot. Let ρ_i denote the position of the ith prismatic actuator relative to the center of mass of the base body in the inertial frame; ρ_i can be expressed as

$$\rho_i = RR_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix}, \quad i = 1, \ldots, n.$$

We summarize the translational and rotational equations of motion in the following; their derivation can be found in the Appendix. Zero linear momentum of the space robot can be written as

$$\begin{aligned} \dot r &= -\frac{\sum_{i=1}^n m_i \dot\rho_i}{M}, \\ &= -\sum_{i=1}^n \frac{m_i}{M} \dot R R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix} - \sum_{i=1}^n \frac{m_i}{M} R R_i \begin{bmatrix} \dot z_i \\ 0 \\ 0 \end{bmatrix}, \end{aligned} \quad (1)$$

where $M = m_b + \sum_{i=1}^n m_i$ is the total mass of the system.

This equation is integrable so that

$$\begin{aligned} r &= r^0 - \frac{\sum_{i=1}^n m_i \rho_i - \sum_{i=1}^n m_i \rho_i^0}{M}, \\ &= r^0 - R \sum_{i=1}^n \frac{m_i}{M} R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix} + R^0 \sum_{i=1}^n \frac{m_i}{M} R_i \begin{bmatrix} z_i^0 \\ l_i \\ 0 \end{bmatrix}, \end{aligned} \quad (2)$$

where r^0, R^0, z_i^0, $i = 1 \cdots n$, are initial values.

Zero angular momentum of the space robot yields

$$\dot R = R \Big\{ \sum_{i=1}^n \widehat{\tilde F_i(z)} \dot z_i \Big\}, \quad (3)$$

where

$$\tilde F_i(z) = J^{-1}(z) F_i(z), \quad i = 1, \cdots, n,$$

and

$$\begin{aligned} J(z) &= I_b + \sum_{i=1}^n \Big(1 - \frac{m_i}{M}\Big) m_i \tilde I_{ii}(z_i) \\ &\quad - \sum_{i=1}^n \sum_{j=1, j\neq i}^n \frac{m_i m_j}{M} \tilde I_{ij}(z_i, z_j), \\ F_i(z) &= -\Big(1 - \frac{m_i}{M}\Big) m_i B_{ii} + \sum_{j=1, j\neq i}^n \frac{m_i m_j}{M} B_{ji}(z_j), \\ B_{ij}(z_i) &= -\widehat{R_j(1)} R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix}, \quad R_j(1) = R_j \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}\tilde{I}_{ij}(z_i, z_j) &= [z_i\ l_i\ 0]R_i^T R_j \begin{bmatrix} z_j \\ l_j \\ 0 \end{bmatrix} I_{3\times 3} \\ &\quad - R_j \begin{bmatrix} z_i z_j & l_i z_j & 0 \\ l_j z_i & l_i l_j & 0 \\ 0 & 0 & 0 \end{bmatrix} R_i^T.\end{aligned}$$

Here the notation $\widehat{a}$, where $a = (a_1, a_2, a_3) \in \mathbb{R}^3$, denotes a skew symmetric matrix given by

$$\widehat{a} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

Therefore, $\widehat{a}b = a \times b$, where $a, b \in \mathbb{R}^3$ and $\times$ is the usual cross product operation.

The above equations of motion completely determine the motion of the space robot. The base body translation and rotation are given by the pair (r, R), and the vector of relative displacements of the actuator masses $z = (z_1, \ldots, z_n)$ define the shape of the space robot.

It can be shown that eqs (1) and (3) can be written in the form

$$\begin{aligned}\begin{bmatrix} v \\ \omega \end{bmatrix} &= -\begin{bmatrix} A_t(z) \\ A_r(z) \end{bmatrix} \dot{z}, \\ \dot{r} &= Rv, \\ \dot{R} &= R\widehat{\omega},\end{aligned}$$

for suitable functions $A_t(z)$ and $A_r(z)$ referred as to the mechanical connection that characterizes the coupling between the shape velocities and linear and angular velocities of the space robot base body expressed in the base body coordinate frame. Rather than using this form in the subsequent development, we make use of eq (3) and the integrated form of eq (1) given in eq (2).

4. Controllability Results

In this section, we carry out controllability analysis for the space robot described by eqs (1) and (2). A variety of control problems can be defined. In particular, we are interested in cases where the number of independent prismatic actuators is less than the six degrees of freedom for the base body position and attitude to be controlled; these are the so-called underactuated cases. These cases lead to non-trivial controllability results and to non-trivial control algorithms.

4.1. Main Results

A simple necessary condition for controllability of base body position and attitude can be obtained immediately. From eq (2), we see that the base body position and attitude are controllable only if there are at least three independent prismatic actuators, i.e., $n \geq 3$, and $[R_i(1)\ R_j(1)\ R_k(1)]$ has full rank, where $R_i(1)$ is the first column of R_i which defines the ith slot axis direction. Otherwise, it is easy to construct a base body position and attitude pair (r, R) (in any neighborhood of any equilibrium) such that no shape variables z can satisfy eq (1), which means that the pair (r, R) cannot be achieved by any shape change. This observation provides a necessary condition on the minimum number of prismatic actuators for controllability and on the design of actuator placement; this condition requires that there are at least three prismatic actuators whose slot axes are linearly independent.

We now study controllability properties for a space robot with three prismatic actuators whose slot axes are linearly independent. The control objective is to achieve desired base body position and attitude changes via actuator mass motions in a given time interval. To simplify the problem, actuator stroke limits are ignored. The equations of motion are given by:

$$r = r^0 - R\sum_{i=1}^{3}\frac{m_i}{M}R_i\begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix} + R^0\sum_{i=1}^{3}\frac{m_i}{M}R_i\begin{bmatrix} z_i^0 \\ l_i \\ 0 \end{bmatrix}, \quad (4)$$

$$\dot{R} = R\Big\{\sum_{i=1}^{3}\widehat{\tilde{F}_i(z)v_i}\Big\}, \quad (5)$$

$$\dot{z}_i = v_i, \quad i = 1, 2, 3, \quad (6)$$

where the actuator velocities defined in eq (6) are viewed as control inputs. We present the main controllability results in the following proposition:

PROPOSITION 1. Under the assumptions that have been introduced, the following statements are equivalent:

1. The base body position and attitude (r, R) are globally controllable;
2. The drift-free control system described by eqs (5) and (6) is small time locally controllable at each $(R, z) \in \mathrm{SO}(3) \times Q_s$;
3. The drift-free control system described by eqs (5) and (6) is small time locally controllable at one shape z for all $R \in \mathrm{SO}(3)$;
4. The Lie algebraic rank condition for the nonlinear drift-free control system described by eqs (5) and (6) holds at one shape;
5. The Lie algebraic rank condition for the nonlinear drift-free control system described by eqs (5) and (6) holds at *every* shape.

Proof. We first show the equivalence between Statements 1 and 2 using eq (4), which can be written as

$$r = RPz + r^0 - R\sum_{i=1}^{3}\frac{m_i}{M}R_i\begin{bmatrix}0\\ l_i\\ 0\end{bmatrix} + R^0\sum_{i=1}^{3}\frac{m_i}{M}R_i\begin{bmatrix}z_i^0\\ l_i\\ 0\end{bmatrix},$$

where $z = [z_1, z_2, z_3]^T$ and

$$P = -\Big[\frac{m_1}{M}R_1(1),\ \frac{m_2}{M}R_2(1),\ \frac{m_3}{M}R_3(1)\Big].$$

Since the three slot axes are linearly independent, P has full rank. Hence, r is uniquely determined by z and vice versa, for any fixed attitude R and for arbitrarily given initial conditions. Now suppose Statement 1 is true, i.e. (r, R) can be globally reached via change of z, it is obvious that arbitrary shape variables z must be achieved for any attitude R. This shows that Statement 1 implies Statement 2. Now we show the other direction. Assume that each (R, z) is controllable by shape change. For arbitrarily given (r, R), we can uniquely determine a corresponding z, and we are able to steer the shape and the attitude to this z and the desired attitude R simultaneously according to the assumption. Then the desired (r, R) is reached. Therefore, arbitrary (r, R) can be achieved. This completes the proof of the equivalence between the first two statements.

We now focus on the equivalence between Statements 2 and 3. Obviously, Statement 2 implies Statement 3; we only need to show the converse. Suppose Statement 3 holds at a shape z^0. That is, there exists a shape control that achieves arbitrary attitude change at z^0 in an arbitrarily given time interval. Let z^1 denote another shape, let $\tilde{R}$ denote an arbitrary attitude change to be achieved at z^1, and $[0, t_f]$ be the given time interval. Let t_1, t_2 satisfy $0 < t_1 < t_2 < t_f$. There exist two suitable finite control inputs, v^1, which steers the shape from z^1 to z^0 in $[0, t_1]$, and v^3, which transfers the shape back to z^1 from z^0 in $[t_2, t_f]$. Furthermore, let R^1 and R^3 denote the unique resulting attitude changes under controls v^1 and v^3, respectively. (Since $\tilde{F}_i(z(t))$, $i = 1, 2, 3$, are finite over any compact time interval, uniqueness follows from the Lipschitz condition.) Note that these attitude changes are independent of the initial attitudes. Consider the following procedure:

Step 1. In the time interval $[0, t_1]$, use v^1 to steer the shape from z^1 to z^0;

Step 2. In the time interval $[t_1, t_2]$, design a control v^2 to achieve the attitude change $(R^1)^{-1}\tilde{R}(R^3)^{-1}$ with zero net change of the shape, i.e. $z(t_2) = z^0$. Since Statement 3 holds at z^0, such v^2 always exists;

Step 3. In the time interval $[t_2, t_f]$, use v^3 to steer the shape from z^0 to z^1.

The above procedure is illustrated schematically in Figure 2.

At the final instant t_f, the final attitude change is given by $R^1(R^1)^{-1}\tilde{R}(R^3)^{-1}R^3 = \tilde{R}$, which is what is desired. This

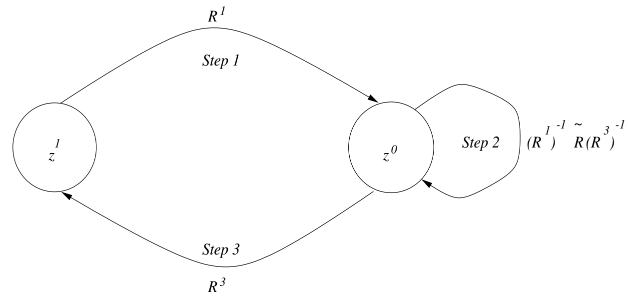

Fig. 2. The schematic shape transfer and attitude change.

means that an arbitrary attitude change can be achieved at z^1 in an arbitrary time interval. Since z^1 is arbitrary, we claim that any (z, R) can be reached as well as its neighborhood. This shows Statement 2 holds.

Next, we show the equivalence between Statements 3 and 4. Since the base body attitude and shape dynamics are described by drift-free eqs (5) and (6), their local controllability is equivalent to local accessibility. It is shown in Rui, Kolmanovsky, and McClamroch (2000) that Statement 4 implies small time local controllability at a designated shape and that arbitrary base body attitude and local shape change can be achieved in a neighborhood of that shape, following the left-invariance property in Rui, Kolmanovsky, and McClamroch (2000) and the fact that SO(3) is compact. This means that Statement 3 holds if Statement 4 is satisfied. We now show the converse. Note that each term in the vector fields $\tilde{F}_i(z)$ has the form $\frac{N(z)}{(\det J(z))^k}$, where $N(z)$ is a polynomial function of z, and $k \in Z^+$. Since $\det J(z) > 0, \forall z$, $\tilde{F}_i(z)$'s are global real analytical functions. Hence, the equations are globally real analytic. Following the argument that the Lie Algebraic Rank Condition (LARC) is necessary for local accessibility for real analytical systems (Sontag 1998), we conclude that Statement 3 implies Statement 4.

We complete the rest of the proof by using the above argument for any shape. Once Statement 2 holds, then controllability holds at *every* shape. Applying the above argument, we have the equivalence between Statements 2 and 5. □

REMARK 1. At least three independent prismatic actuators are required to achieve local controllability of the space robot base body translation and rotation. Further, three independent prismatic actuators can achieve "global" controllability of the space robot base body translation and rotation if attitude and local shape controllability conditions are satisfied at some shape.

REMARK 2. Proposition 1 expresses the equivalence between the controllability of the base body translation and rotation and the controllability of the base body attitude and shape. This equivalence property can be used to simplify the controllability tests.

REMARK 3. If stroke limits are placed on the displacement of the actuator masses, then the translational control authority of the base body is substantially limited, but the rotational control authority is not limited in the sense that arbitrary base body attitude maneuvers can be achieved at any shape satisfying the constraint.

REMARK 4. A local controllability test for eqs (4) and (5) has been given in Rui, Kolmanovsky, and McClamroch (2000). This test provides a sufficient condition for global attitude and local shape change based on the LARC, which can be checked by computing Lie brackets. The first order Lie brackets for vector fields $\tilde{F}_i(z)$ and $\tilde{F}_j(z)$, $i, j = 1, 2, 3, i \neq j$, are given by

$$\omega_{[\tilde{F}_i(z),\tilde{F}_j(z)]} = \tilde{F}_j(z) \times \tilde{F}_i(z) + \frac{\partial \tilde{F}_i(z)}{\partial z_j} - \frac{\partial \tilde{F}_j(z)}{\partial z_i}, \tag{7}$$

with higher order Lie brackets defined accordingly. An easily computed formula is given by

$$\begin{aligned}\omega_{[\tilde{F}_i(z),\tilde{F}_j(z)]} &= \frac{J(z)}{\det J(z)}[F_j(z) \times F_i(z)] \\ &\quad -J^{-1}(z)\left[\frac{\partial J(z)}{\partial z_j} J^{-1}(z) F_i(z) \right. \\ &\quad \left. -\frac{\partial J(z)}{\partial z_i} J^{-1}(z) F_j(z)\right] \\ &\quad +J^{-1}(z)\left[\frac{\partial F_i(z)}{\partial z_j} - \frac{\partial F_j(z)}{\partial z_i}\right]. \end{aligned} \tag{8}$$

The derivation of this formula is given in the Appendix.

REMARK 5. According to Proposition 1, controllability may be verified at any base body attitude and corresponding shape. The controllability results presented in Proposition 1 simplify the test considerably. It is not necessary to check the LARC at every shape; one only needs to find one shape where the LARC holds. A shape may be chosen that leads to relatively simple Lie bracket computations.

REMARK 6. Since local controllability implies global controllability, we will make no further distinction between local and global controllability in the subsequent development.

REMARK 7. Once controllability is verified, Statement 5 in Proposition 1 implies that there always exist Lie brackets that span the tangent space of SO(3) at each shape. Therefore, control algorithms can be developed based on Lie bracket expansions using any set of spanning Lie brackets. Since different choices of spanning Lie brackets correspond to different algorithms with different computational complexity, the choice can be made to simplify the motion planning algorithms and to reduce the computational complexity.

PROPOSITION 2. If there are three Lie brackets that satisfy the LARC at some shape, then these Lie brackets satisfy the LARC on an open dense subset of the shape space.

Proof. Suppose that there are three Lie brackets satisfying the LARC at shape z^0. Recall that the vector fields $\tilde{F}_i(z)$ are globally real analytic; therefore the three Lie brackets are also global analytic functions of the shape z, because each term of the brackets is a rational function of the form $\frac{P(z)}{(\det J(z))^j}$, where $P(z)$ is a polynomial function of z, $j \in Z^+$, and $\det J(z) > 0, \forall z$. Hence, the determinant $d_\omega(z)$ of the matrix formed by the three Lie brackets is a global analytical function of z, which satisfies $d_\omega(z^0) \neq 0$. It is easy to see that the set satisfying $d_\omega(z) \neq 0$ is open in the shape space; we only need to show that this open set is dense, that is, its closure is the whole shape space. An equivalent argument is that every neighborhood of the shape with zero d_ω contains a shape with nonzero d_ω (Munkres 1974). Suppose not. This means that there is an open set in which d_ω is identically zero. Since $d_\omega(z)$ is globally real analytical and the shape space is connected, $d_\omega(z)$ is identically zero on the whole shape space (Courant and John 1965). This contradicts the condition that $d_\omega(z^0) \neq 0$. □

This proposition can be interpreted as follows. Suppose we find three Lie brackets spanning the tangent space of SO(3) at one shape. According to Proposition 2, the shape where the LARC fails cannot be clustered as an open ball in the three-dimensional shape space. This implies that at almost all shapes, these Lie brackets span the tangent space. Therefore, a shape change algorithm constructed based on these particular spanning Lie brackets applies to almost every shape. Even if the LARC fails at some shape, a nearby shape can be located where the LARC holds, and the procedure described in the proof of Proposition 1 can be used to achieve the desired motion. This means that a single algorithm can be constructed to solve motion planning problems at different shapes. Of course, there may be many Lie bracket combinations satisfying the LARC, but we can choose the simplest one, e.g., using only the first order Lie brackets, to reduce computational complexity.

In the following, we present an example to show that if all the three slots have zero offsets, then the space robot base body position and attitude are controllable via shape change. We further show a stronger result, i.e., for almost all offsets, the space robot base body position and attitude are controllable.

4.2. A Space Robot Controlled by Three Prismatic Actuators: Controllability Analysis

We check the controllability for a space robot with three independent prismatic actuators whose slot axes have zero offsets, i.e., $l_1 = l_2 = l_3 = 0$. In such a case, we have

$$F_1(z) = \frac{m_1 m_2 z_2}{M} R_2(1) \times R_1(1) + \frac{m_1 m_3 z_3}{M} R_3(1) \times R_1(1),$$
$$F_2(z) = \frac{m_2 m_1 z_1}{M} R_1(1) \times R_2(1) + \frac{m_2 m_3 z_3}{M} R_3(1) \times R_2(1),$$

$$F_3(z) = \frac{m_3 m_1 z_1}{M} R_1(1) \times R_3(1) + \frac{m_3 m_2 z_2}{M} R_2(1) \times R_3(1).$$

We first check local controllability at the origin $z = (0, 0, 0)$ by computing the first order Lie brackets. At $z = (0, 0, 0)$, it is easy to show that $F_i(z) = 0$ and

$$\frac{\partial F_i(z)}{\partial z_j} = \frac{m_i m_j}{M} R_j(1) \times R_i(1).$$

Therefore, using eq (8), the first order Lie bracket $\omega_{[\tilde{F}_i(z), \tilde{F}_j(z)]}$ evaluated at $z = (0, 0, 0)$ can be expressed as

$$\omega_{[\tilde{F}_i(z), \tilde{F}_j(z)]}\Big|_{z=0} = J^{-1}(0) \frac{2 m_i m_j}{M} R_j(1) \times R_i(1).$$

The three first order Lie brackets, evaluated at $z = (0, 0, 0)$, form the following matrix:

$$\begin{aligned} \tilde{G} &= \Big[\omega_{[\tilde{F}_1(z), \tilde{F}_2(z)]}, \omega_{[\tilde{F}_1(z), \tilde{F}_3(z)]}, \omega_{[\tilde{F}_2(z), \tilde{F}_3(z)]}\Big]_{z=0} \\ &= \frac{2J^{-1}(0)}{M} \Big[m_1 m_2 R_2(1) \times R_1(1), \\ &\qquad m_1 m_3 R_3(1) \times R_1(1),\ m_2 m_3 R_3(1) \times R_2(1)\Big]. \end{aligned}$$

Since $[R_1(1), R_2(1), R_3(1)]$ are linearly independent, it can be shown that

$$\Big[m_1 m_2 R_2(1) \times R_1(1),\ m_1 m_3 R_3(1) \times R_1(1),\ m_2 m_3 R_3(1) \times R_2(1)\Big]$$

has full rank, which implies that the three first order Lie brackets span the tangent space of SO(3) at $z = (0, 0, 0)$. As a result, eqs (4) and (5) satisfy the LARC at $z = (0, 0, 0)$. Using Proposition 1, it is clear that the system is controllable at *every* shape, and the base body position and attitude are controllable via shape change.

It is interesting to note that the determinant function d_ω in the proof of Proposition 2 can be viewed as a global real analytic function not only of the shape $z = (z_1, z_2, z_3)$ but also of the offset vector $l = (l_1, l_2, l_3)$. Since $d_\omega \neq 0$ at $z = (0, 0, 0)$ and $l = (0, 0, 0)$ as shown above, we see that $d_\omega|_{z=(0,0,0)} \neq 0$ on an open dense subset of the offset vector $l = (l_1, l_2, l_3)$, following the argument in the proof of Proposition 2. This implies that the first order Lie brackets satisfy the LARC at $z = (0, 0, 0)$ for almost all offset combinations. Hence, using Proposition 2, we further conclude:

PROPOSITION 3. The space robot base body position and attitude are controllable for almost all offset combinations.

5. Design of Space Robot Maneuvers

In this section, we present a motion planning procedure based on the controllability results developed previously, assuming controllability is verified. The objective is to achieve a desired space robot base body position and attitude change via prismatic actuator motions. The basic idea follows (Shen and McClamroch 2001): steer the actuator masses to their desired final shape which can be uniquely determined by solving a simple set of algebraic equations; then use periodic shape motions to achieve the desired attitude of the base body. Control algorithms based on Lie bracket expansions are constructed to obtain the desired shape motions. The construction follows the development in Rui, Kolmanovsky, and McClamroch (2000).

We describe the maneuver design procedure as follows. Assume that the base body initial position and attitude r^0, R^0 are to be transferred to final position and attitude r^f, R^f in the given time interval $[0, t_f]$; the initial shape is z^0.

Step 1. Determine the unique values of the final shape vector z^f that solves the algebraic equation

$$r^f - r^0 = -R^f \sum_{i=1}^{3} \frac{m_i}{M} R_i \begin{bmatrix} z_i^f \\ l_i \\ 0 \end{bmatrix} + R^0 \sum_{i=1}^{3} \frac{m_i}{M} R_i \begin{bmatrix} z_i^0 \\ l_i \\ 0 \end{bmatrix}. \quad (9)$$

Step 2. Steer the shape variables from z^0 to z^f in $[0, t_1]$, where $0 < t_1 < t_f$. A suitable choice for the shape velocities is

$$v_i(t) = c_i \sin^2\left(\frac{2\pi n_1 t}{t_1}\right), \quad i = 1, 2, 3, \quad (10)$$

where $n_1 \in Z^+$, and $c_i = \frac{2(z_i^f - z_i^0)}{t_1}$. This necessarily guarantees $z_i(t_1) = z_i^f$ and $\dot{z}_i(0) = \dot{z}_i(t_1) = 0$.

Step 3. Design a periodic shape change that steers the base body attitude to R^f in $[t_1, t_f]$ while leading to zero net change in the shape, so that the final shape remains z^f. Since the space robot is controllable by shape change, such inputs always exist. To be more specific, we consider a first order Lie bracket based algorithm in Rui, Kolmanovsky, and McClamroch (2000) for illustration. We assume satisfaction of the LARC at z^f, that is the first order Lie brackets $\omega_{[\tilde{F}_1(z), \tilde{F}_2(z)]}$, $\omega_{[\tilde{F}_1(z), \tilde{F}_3(z)]}$, and $\omega_{[\tilde{F}_2(z), \tilde{F}_3(z)]}$ at z^f are linearly independent.

Find $\omega_d \in \mathbb{R}^3$ satisfying

$$R^{-1}(t_1) R^f = e^{\widehat{\omega_d}},$$

where $R(t_1)$ is the base body attitude at the end of Step 2. Determine $\alpha \in \mathbb{R}^3$ as

$$\begin{aligned} \alpha &= [\alpha_1,\ \alpha_2,\ \alpha_3]^T \\ &= \Big[\omega_{[\tilde{F}_1(z), \tilde{F}_2(z)]}, \omega_{[\tilde{F}_2(z), \tilde{F}_3(z)]}, \omega_{[\tilde{F}_1(z), \tilde{F}_3(z)]}\Big]_{z=z^f}^{-1} \omega_d. \end{aligned}$$

Choose $n \in Z^+$ and shape velocity inputs in $[t_1, t_f]$ as

$$\begin{aligned} v_1(t) &= \frac{2\pi\sqrt{n}}{t_f - t_1}\Big(b_{11}\sin\frac{2\pi n(t-t_1)}{t_f-t_1} \\ &\quad + b_{12}\sin\frac{4\pi n(t-t_1)}{t_f-t_1}\Big), \\ v_2(t) &= \frac{2\pi\sqrt{n}}{t_f - t_1}\Big(b_{21}\sin\frac{2\pi n(t-t_1)}{t_f-t_1} \\ &\quad + a_{22}\cos\frac{4\pi n(t-t_1)}{t_f-t_1} - a_{22}\cos\frac{6\pi n(t-t_1)}{t_f-t_1}\Big), \\ v_3(t) &= \frac{2\pi\sqrt{n}}{t_f - t_1}\Big(a_{31}\cos\frac{2\pi n(t-t_1)}{t_f-t_1} \\ &\quad - a_{31}\cos\frac{6\pi n(t-t_1)}{t_f-t_1}\Big), \end{aligned}$$

where the parameters $b_{11},\ b_{12},\ b_{21},\ a_{22}$, and a_{31} satisfy

$$a_{22}b_{12} = \frac{2\alpha_1}{\pi},\ a_{31}b_{21} = \frac{\alpha_2}{\pi},\ a_{31}b_{11} = \frac{\alpha_3}{\pi}.$$

This completes the development of the maneuver design procedure. It can be verified that $\dot{z}_i(t_1) = \dot{z}_i(t_f) = 0,\ i = 1, 2, 3.$ An attitude error bound for this algorithm is given in Rui, Kolmanovsky, and McClamroch (2000).

We now provide an example to illustrate the maneuver design procedure described above. The space robot consists of a rigid base body and three independent prismatic actuators, whose slot axes pass through the center of mass the base body and each slot axis is aligned with a principal axis of the base body. Therefore, the offsets $l_i = 0,\ i = 1, 2, 3$, and $[R_1(1)\ R_2(1)\ R_3(1)]$ is the 3×3 identity matrix.

The mass for the base body and the actuator masses are $m_b = 10$ kg, and $m_1 = m_2 = m_3 = 2$ kg; the base body inertia is given by $I_b = \text{diag}(1, 1.5, 1.5)\text{kgm}^2$. At the initial instant $t = 0$, $r^0 = 0$, $R^0 = I_3$, and $z^0 = (0, 0, 0)$. The maneuver task is to transfer the base body from the given initial position and attitude to the final position $r^f = (0.3, -0.25, 0.1)$ m and final attitude expressed in exponential coordinates as $R^f = e^{\widehat{\omega^f}}$ in 4900 s, where $\omega^f = (0.5, -0.15, 0.1)$ rad.

Based on the controllability results in Section 4.2, we know that the space robot is controllable via shape change. We design the required shape motions, following the indicated procedure. First, we determine the required final shape z^f from eq (8); the result is $z^f = (-2.3784, 1.6873, -1.3377)$ m.

In the second step, the robot shape is changed from z^0 to z^f in the first 100 s. Following Step 2 in the procedure, the shape velocities have the form given in eq (9), where the parameters are chosen as $n_1 = 10$, and $c_1 = -0.0476,\ c_2 = 0.0337,\ c_3 = -0.0275$. At $t = 100$ s, the robot base body attitude is almost unchanged, i.e., $R(100) = I_3$.

In the third step, we construct periodic shape motions to achieve the desired final robot base body attitude while keeping the shape unchanged at the final time. The three first-order Lie brackets at z^f span the tangent space of SO(3) as shown in Section 4.2. Hence, we solve for $\omega_2 \in \mathbb{R}^3$ satisfying $e^{\widehat{\omega_2}} = R^{-1}(100)R^f$; the result is $\omega_2 = (0.5, -0.15, 0.1)$. Next we determine α as

$$\begin{aligned} \alpha &= \Big[\omega_{[\tilde{F}_1(z),\tilde{F}_2(z)]},\ \omega_{[\tilde{F}_1(z),\tilde{F}_3(z)]},\ \omega_{[\tilde{F}_2(z),\tilde{F}_3(z)]}\Big]^{-1}_{z=z^f}\omega_2 \\ &= [0.8438, -61.09, -18.82]^T. \end{aligned}$$

As a result, we choose the periodic shape velocities in $t \in [100, 4900]$ as

$$\begin{aligned} v_1(t) &= 0.05236\Big(-0.7373\sin\frac{2\pi(t-100)}{3} \\ &\quad -0.6715\sin\frac{4\pi(t-100)}{3}\Big), \\ v_2(t) &= 0.05236\Big(-2.393\sin\frac{2\pi(t-100)}{3} \\ &\quad +0.8\cos\frac{4\pi(t-100)}{3} - 0.8\cos 2\pi(t-100)\Big), \\ v_3(t) &= 0.05236\Big(8.125\cos\frac{2\pi(t-100)}{3} \\ &\quad -8.125\cos 2\pi(t-100)\Big). \end{aligned}$$

This completes the maneuver construction.

MATLAB has been used to simulate the space robot motions corresponding to this maneuver. The simulation results for the robot base body position and attitude as well as for the shape are shown in Figures 3–5, and the space robot configuration changes are presented in Figures 6–8. At the final instant $t_f = 4900$ s, the base body attitude in exponential coordinates is given by $\omega(t_f) = (0.5342, -0.1511, 0.0943)$ rad, and the base body position is $r(t_f) = (0.2995, -0.2557, 0.0931)$ m. The small errors arise from the Lie bracket expansion approximation in the third step of the control algorithm.

6. Conclusions

We have studied the dynamics and control of a specific underactuated space robot, focusing on three dimensional maneuvers using prismatic actuators. The feasibility of using such shape change mechanisms to simultaneously control space robot base body translational and rotational motions has been demonstrated. Nonlinear control theory has been used for controllability analysis and for development of rest-to-rest space robot maneuvers. These results can be applied to advanced space robot missions.

In this paper, all gravitational and gravity gradient effects are ignored; such an approximation is often justified. Our ongoing research is concerned with the case that gravity gradient effects are included in the model and utilized to achieve base body control via shape change.

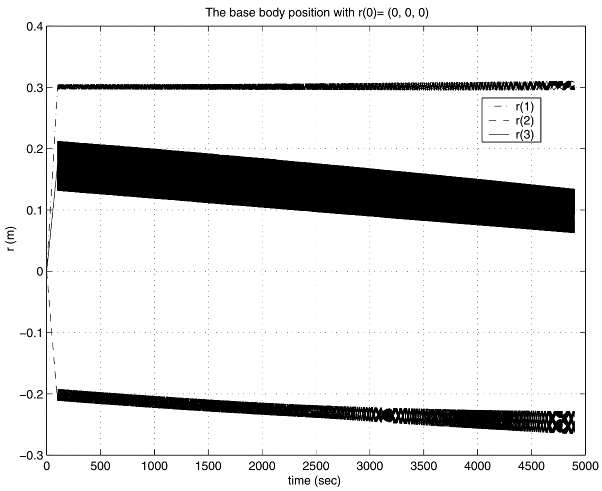

Fig. 3. Time response of the base body position.

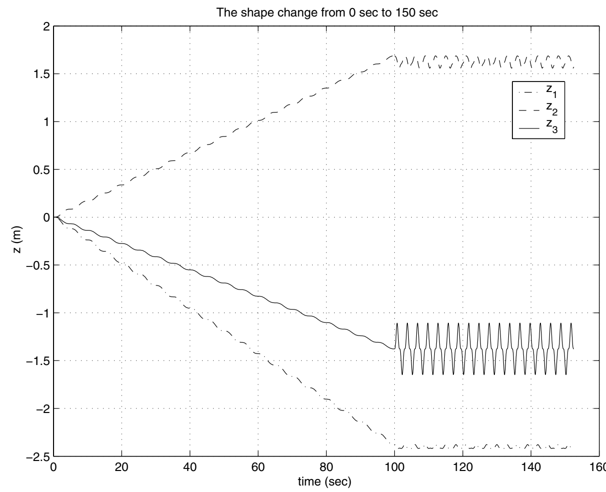

Fig. 5. Time response of the shape change in the first 150 s.

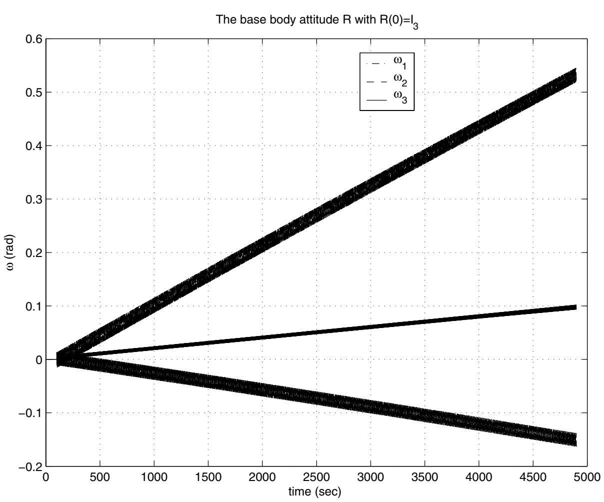

Fig. 4. Time response of the base body attitude, expressed in exponential coordinates.

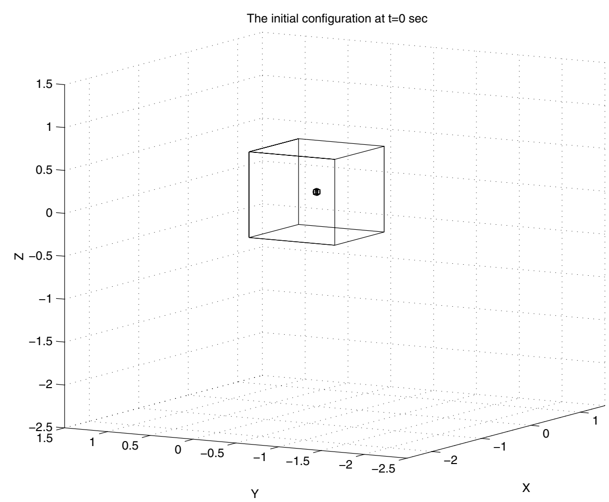

Fig. 6. Initial space robot configuration at $t = 0$ s. "+": the origin of the inertial frame (coincides with the initial position of the center of mass of the base body); "○": the center of mass of the base body.

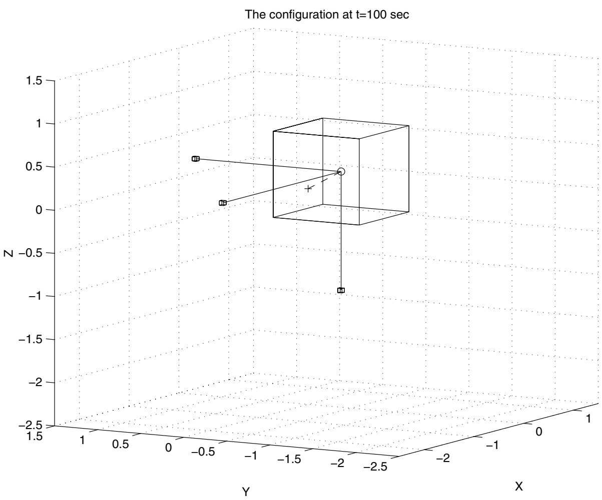

Fig. 7. Space robot configuration at $t = 100$ s.

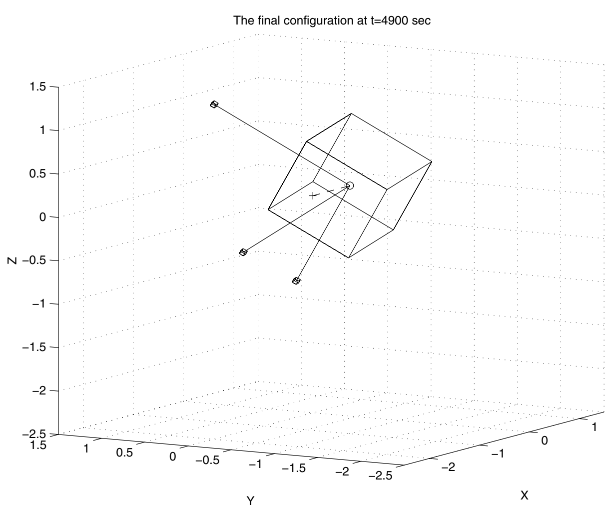

Fig. 8. Final space robot configuration at $t = 4900$ s.

Appendix

A.1. Derivation of the Translation and Rotation Equations of Motion

We derive the equations of motion for a space robot with n prismatic actuators. An inertial frame is chosen at the fixed center of mass of the space robot. Let ρ_i denote the position of the ith actuator mass relative to the center of mass of the base body in the inertial frame; then ρ_i can be expressed as

$$\rho_i = RR_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix}, \quad i = 1, \ldots, n.$$

As shown before, the assumption that the total linear momentum is zero implies

$$\dot{r} = -\sum_{i=1}^{n} \frac{m_i}{M} \dot{R} R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix} - \sum_{i=1}^{n} \frac{m_i}{M} R R_i \begin{bmatrix} \dot{z}_i \\ 0 \\ 0 \end{bmatrix}, \quad \text{(A1)}$$

where $M = m_b + \sum_{i=1}^{n} m_i$ is the total mass of the system.

Let ρ_i^B be the relative position of the ith actuator mass given by

$$\rho_i^B = R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix},$$

and ω be the angular velocity of the base body satisfying $\dot{R} = R\widehat{\omega}$, both expressed in the base body frame. The assumption that the total angular momentum is zero implies

$$H_b + \sum_{i=1}^{n} H_i = 0, \quad \text{(A2)}$$

where H_b denotes the angular momentum of the base body and H_i represents the angular momentum of the ith prismatic given by

$$H_b = m_b r \times \dot{r} + R I_b \omega, \quad \text{(A3)}$$

$$H_i = m_i (r + \rho_i) \times (\dot{r} + \dot{\rho}_i). \quad \text{(A4)}$$

Substituting eqs (A3) and (A4) into (A2), and making use of the linear momentum eq (A1), we have

$$-\frac{1}{M} \sum_{i=1}^{n} m_i \rho_i^B \times \sum_{i=1}^{n} m_i (\dot{\rho}_i)^B + \sum_{i=1}^{n} m_i \rho_i^B \times (\dot{\rho}_i)^B + I_b \omega = 0,$$

where $(\dot{\rho}_i)^B = R^{-1} \dot{\rho}_i$ is the relative velocity of the ith actuator mass expressed in the body frame.

Since

$$(\dot{\rho}_i)^B = \dot{\rho}_i^B + \omega \times \rho_i^B,$$

we obtain

$$\rho_i^B \times (\dot{\rho}_j)^B = B_{ij}(z_i)\dot{z}_j + \tilde{I}_{ij}(z_i, z_j)\omega,$$

where $B_{ij} \in \mathbb{R}^3$ and $\tilde{I}_{ij}(z_i) \in \mathbb{R}^{3\times 3}$ are given by

$$\begin{aligned} B_{ij}(z_i) &= -\widehat{R_j(1)}R_i \begin{bmatrix} z_i \\ l_i \\ 0 \end{bmatrix}, \quad R_j(1) = R_j \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ \tilde{I}_{ij}(z_i, z_j) &= [z_i \; l_i \; 0]R_i^T R_j \begin{bmatrix} z_j \\ l_j \\ 0 \end{bmatrix} I_{3\times 3} \\ &\quad -R_j \begin{bmatrix} z_i z_j & l_i z_j & 0 \\ l_j z_i & l_i l_j & 0 \\ 0 & 0 & 0 \end{bmatrix} R_i^T. \end{aligned}$$

If $i = j$, we get

$$\rho_i^B \times (\dot{\rho}_i)^B = B_{ii}\dot{z}_i + \tilde{I}_{ii}(z_i)\omega,$$

where

$$\begin{aligned} B_{ii} &= R_i \begin{bmatrix} 0 \\ 0 \\ -l_i \end{bmatrix}, \quad \tilde{I}_{ii}(z_i) = (z_i^2 + l_i^2)I_{3\times 3} \\ &\quad -R_i \begin{bmatrix} z_i^2 & l_i z_i & 0 \\ l_i z_i & l_i^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_i^T. \end{aligned}$$

It is easy to verify that $\tilde{I}_{ii}(z_i)$ is positive semi-definite for all z_i, l_i.

Finally, we obtain the angular momentum equation

$$J(z)\omega = \sum_{i=1}^{n} F_i(z)\dot{z}_i, \tag{A5}$$

where

$$\begin{aligned} J(z) &= I_b + \sum_{i=1}^{n}(1 - \frac{m_i}{M})m_i \tilde{I}_{ii}(z_i) \\ &\quad - \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \frac{m_i m_j}{M} \tilde{I}_{ij}(z_i, z_j), \\ F_i(z) &= -\left(1 - \frac{m_i}{M}\right) m_i B_{ii} + \sum_{j=1, j\neq i}^{n} \frac{m_i m_j}{M} B_{ji}(z_j). \end{aligned}$$

It can be shown that $J(z)$ is positive definite for all z.

Consequently, the rotation equations can be written as

$$\dot{R} = R\Big\{\sum_{i=1}^{n} \widehat{\tilde{F}_i(z)\dot{z}_i}\Big\}, \tag{A6}$$

where

$$\tilde{F}_i(z) = J^{-1}(z)F_i(z), \quad i = 1, \ldots, n.$$

Note that rotational eq (A6) is of the same form as the rotational equations for a multibody space system with n appendages given in Rui, Kolmanovsky, and McClamroch (2000).

A.2. Derivation of Equation (8)

To rewrite the Lie bracket expression, the following identity is used:

$$\frac{\partial J^{-1}(z)}{\partial z_j} = -J^{-1}(z)\frac{\partial J(z)}{\partial z_j}J^{-1}(z).$$

Since $J(z)$ is positive definite for all z, it can be decomposed as $J(z) = R^T \Lambda R$, where R is orthogonal, and $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$, $\lambda_i > 0$. Hence,

$$\begin{aligned} \tilde{F}_i(z) \times \tilde{F}_j(z) &= J^{-1}(z)F_i(z) \times J^{-1}(z)F_j(z), \\ &= R^T\Big(\Lambda^{-1}RF_i(z)\Big) \times R^T\Big(\Lambda^{-1}RF_j(z)\Big), \\ &= R^T\Big(\Lambda^{-1}RF_i(z) \times \Lambda^{-1}RF_j(z)\Big), \\ &= \frac{R^T\Lambda}{\lambda_1\lambda_2\lambda_3}\Big(RF_i(z) \times RF_j(z)\Big), \\ &= \frac{R^T\Lambda R}{\lambda_1\lambda_2\lambda_3}\Big(F_i(z) \times F_j(z)\Big), \\ &= \frac{J(z)}{\det J(z)}\Big(F_i(z) \times F_j(z)\Big), \end{aligned}$$

where we use $\det J(z) = \lambda_1\lambda_2\lambda_3$. Equation (8) is then obtained by substituting the above identities into the original expression.

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References

Arai, H., Tanie, K., and Tachi, S. 1993. Dynamic control of a manipulator with passive joints in operational space. *IEEE Transactions on Robotics and Automation* 9(1):85–93.

Arai, H., Tanie, K., and Shiroma, N. 1998. Nonholonomic control of a three-DoF planar underactuated manipulator. *IEEE Transactions on Robotics and Automation* 14(5):681–695.

Carignan, C. R., and Akin, D. L. 2000. The reaction stabilization of on-orbit robots. *IEEE Control Systems Magazine* 20(6):19–33.

Courant, R., and John, F. 1965. *Introduction to Calculus and Analysis*, vol. 1, pp. 544–545. Interscience Publishers.

Di Giamberardino, P., and Monaco, S., and Normand-Cyrot, D. 2000. Hybrid control scheme for maneuvering space multibody structures. *AIAA Journal of Guidance, Control, and Dynamics* 23(2):231–240.

Kelly, S., and Murray, R. M. 1995. Geometric phases and robotic locomotion. *Journal of Robotic Systems* 12(6):417–431.

Kolmanovsky, I., and McClamroch, N. H. 1995. Developments in nonholonomic control problems. *IEEE Control Systems Magazine* 15(6):20–36.

Koningstein, R., and Cannon, R. H. Jr. 1995. Experiments with model-simplified computed-torque manipulator controllers for free-flying robots. *AIAA Journal of Guidance, Control and Dynamics* 18(6):1386–1390.

Krishnaprasad, P. S. 1990. Geometric phases, and optimal reconfiguration. *Proceedings of the American Control Conference*, pp. 2440–2444.

Leonard, N. E., and Krishnaprasad, P. S. 1995. Motion control of drift-free left-invariant systems on Lie groups. *IEEE Transactions on Automatic Control* 40(9):1539–1554.

Li, Z. X., and Cannay, J. F. 1990. Motion of two rigid objects with rolling constraints. *IEEE Transaction on Robotics and Automation*, vol. RA2-06, pp. 62–72.

Matsumoto, S., Wakabayashi, Y., and Watanbe, Y. 2000. Attitude control for free-flying space robot using gravity gradient torque and manipulator posture. *Proceedings of 2000 AIAA Guidance, Navigation, and Control Conference and Exhibit*. Denver, CO.

Munkres, J. R. 1974. *Topology: A First Course*, pp. 94–95. Prentice Hall Inc.

Murray, R. M., Li, Z., and Sastry, S. 1994. *A Mathematical Introduction to Robotic Manipulation*. CRC Press.

Murray, R. M., and Sastry, S. S. 1993. Nonholonomic motion planning: Steering using sinusoids. *IEEE Transactions on Automatic Control* 38(5):700–716.

Reyhanoglu, M., and McClamroch, N. H. 1992. Reorientation maneuvers of planar multi-body systems in space using internal controls. *AIAA Journal of Guidance, Control and Dynamics* 15(6):1475–1480.

Rui, C., Kolmanovsky, I., and McClamroch, N. H. 2000. Nonlinear attitude and shape control of spacecraft with articulated appendages and reaction wheels. *IEEE Transactions on Automatic Control* 45(8):1455–1469.

Senda, K., Murotsu, Y., Nagaoka, H., and Mitsuya, A. 1995. Attitude control for free-flying space robot with CMG, *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Baltimore, MD, pp. 1494–1502.

Shen, J., and McClamroch, N. H. 2001. Translational and rotational spacecraft maneuvers via shape change actuators. *Proceedings of the American Control Conference*, Arlington, VA, pp. 3961–3966.

Sontag, E. D. 1998. *Mathematical Control Theory: Deterministic Finite Dimensional Systems*, 2nd ed., pp. 177–179. Springer-Verlag.

Spofford, J. R., and Akin, D. L. 1990. Redundancy control of a free-flying Telerobot. *AIAA Journal of Guidance, Control and Dynamics* 13(3):515–523.

Sreenath, N. 1992. Nonlinear control of planar multibody systems in shape space. *Mathematics of Control, Signals, and Systems*, vol. 5, pp. 343–363.

Ueba, M., and Yasaka, T. 1994. Attitude maneuver of service vehicle with spinning spent satellite. *AIAA Journal of Guidance, Control and Dynamics* 17(5):1007–1010.

Umetani, Y., and Yoshida, K. 1989. Resolved motion rate control of space manipulator with generalized Jacobian matrix. *IEEE Transactions on Robotics and Automation* 5(3):303–314.

Walsh, G. C., and Sastry, S. S. 1995. On reorienting linked rigid bodies using internal motions. *IEEE Transactions on Robotics and Automation* 11(1):139–146.

Yoshida, K., and Umetani, Y. 1990. Control of space free-flying robot. *Proceedings of the 29th IEEE Conference on Decision and Control*, Honolulu, Hawaii, pp. 97–102.